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APPLICATION OF DIFFERENCE EQUATIONS IN DC CIRCUITS

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Highlights:

- *Demonstrates the role of difference equations in modelling electrical circuit behaviour.*
- *Highlights how difference equations capture transient and steady-state responses.*
- *Establishes difference equations as practical tools in electrical engineering analysis, with clarified scalability and stability.*

Abstract: This study explores the application of difference equations in modelling and analysing electrical circuits, specifically 4-mesh DC, RL, and RLC topologies. With the increasing integration of discrete-time simulation and digital control in engineering systems, traditional methods such as mesh analysis may be limited by computational complexity. Difference equations offer a discrete framework to simulate circuit dynamics using time-stepped recurrence relations. First- and second-order difference equations are applied and derived from RL and RLC circuit models in continuous time using discrete-time approximations. Solution of ladder circuits is through characteristic equation for second-order recurrence relations, whereas recursive formulations are adopted to analyse transient responses in RL and RLC circuits. The study defines complexity in terms of equation count and compares it across methods to demonstrate improved scalability. Realistic component values are selected to reflect conditions in actual electrical systems, such as inductive startup loads and resonant filter behaviour. The resulting current responses demonstrate convergence and system stabilization over time. Stability is confirmed analytically through characteristic roots and time-step considerations. These visualizations reinforce the suitability of difference equations for modelling dynamic responses in power electronics, sensor systems, and digital control applications. The findings highlight how difference equations provide a viable alternative for efficient and scalable analysis in modern electrical engineering.

Keywords: Difference Equations; Electrical Circuits; Ladder Circuits; RL Circuit; RLC Circuit

1. Introduction

Direct Current (DC) circuit analysis is important in Electrical Engineering. It establishes fundamental principles in designing unidirectional current systems. The principles in DC circuit analysis serve as the foundation for enhancing performance and having the safety of solar panel installations and electric vehicles maintained (Bigelow, 2020). It encloses the principles behind many electrical components including resistive circuits, capacitors, and magnetic circuits. These provide a foundation for understanding how electric currents behave in various systems aiding electrical engineers. It explains how resistors control current flow, how capacitors and inductors store and release energy, and explains how magnetic fields interact with electric circuits (Sedra *et al.*, 2020). Having these concepts mastered is needed for anyone seeking to analyze, design, and troubleshoot electrical systems safely and effectively (Fiore, 2020).

Scholars that desire to tackle advanced electrical systems must first understand DC circuit analysis. DC analysis principles allow modern electrical engineers to create simulation tools and software (Rahmani-Andebili, 2020). The analysis of electrical circuits heavily depends on three fundamental network components including Resistor-Inductor (RL) and Resistor-Inductor-Capacitor (RLC) along with ladder networks (Alexander & Sadiku, 2021). The analysis of transient behavior in electrical systems depends on RL circuits because they demonstrate fundamental understanding of systems with significant inductive elements. RLC circuits that can combine both inductance and capacitance are important components for the analysis of resonance behavior and filtering systems.

The structure of ladder networks that are repetitive in nature makes them suitable for digital-to-analog converter as well as filter designs. The time-domain examination of fractional electrical circuits also works with ladder elements to model complex systems (Piotrowska & Rogowski, 2021). In deriving generalized formulas that can simplify the computation of electrical properties in homogeneous ladder networks, difference equations play a vital role in allowing for a more efficient circuit modeling and analysis (Konjeti & Mondal, 2023).

A difference equation is used to relate successive members of a series and differences (Leydold, 2024). It is usually recursive, and it allows computing the outputs of a computing system based on their inputs and their preceding outputs, and it can be applied to computing the transform of a system (Ahmed *et al.*, 2022). Being the discrete analog of the differential equations, it propagates functions on discrete steps, and the interval between them is typically constant such as one (Weisstein, 2025; The Editors of Encyclopaedia Britannica, 2025).

The first-order equations have dependence on a term preceding it and potentially a constant resulting in simple linear recurrences, and the second-order equations have two terms preceding them and a higher degree of complexity (Presa *et al.*, 2025; Hammond, 2020). The behavior of the system after each initial condition and coefficients is either convergent, oscillatory, or divergent (Hammond, 2020; Presa et al., 2025). These are referred to as dynamic models, and these equations are commonly applied in fields such as population growth and signal processing (Green, 2024; Kavitha & Raj, 2024).

1.1. Research Objectives

The widespread accessibility of mesh analysis for DC circuit analysis has not eliminated the necessity to develop alternative methods that better suit discrete-time modeling and digital simulation. The accurate representation of electrical circuits under sampled data conditions requires different equations because modern electrical systems increasingly use digital control strategies. The paper focuses on the application of difference equations for electrical circuits. This paper demonstrates the advantages of using difference equations instead of traditional methods. More specifically, it aims to:

1. To apply difference equations in the analysis of ladder, RL, and RLC circuits using discrete-time formulations.

2. To highlight the relevance of difference equations in modern engineering systems, particularly in digital control, sensor networks, and power electronics, through analysis of transient response and system behaviour.

3. To compare the complexity of difference equation and mesh methods in terms of symbolic equation count and assess stability through characteristic analysis.

1.2. Introduction to First-Order Difference Equation

First-order linear difference equations can be solved using a method similar to that used for linear differential equations. Consider the initial value problem:

$$X_{k+1} = \alpha X_k + \beta \tag{1}$$

$$X_0 = 1, \tag{2}$$

Here, α is a constant. This qualifies as an initial value problem since the starting term x_0 is given. The general solution is obtained by combining the homogeneous and particular solutions: where α is a fixed constant.

$$X_k = H_k + P_k \tag{3}$$

The homogeneous component satisfies the associated homogeneous equation: X_{k+1} = αX_k. It is evident that the solution to this homogeneous equation is a constant sequence scaled by k, expressed as: H_k = $\alpha^k H_0$ where H_0 is an arbitrary constant to be determined later by initial conditions.

Next, a particular solution P_k must satisfy:

$$P_{k+1} = \alpha P_k + \beta \tag{4}$$

Assuming a constant solution P_k = C, substitution yields. Then we have C = αC + β, or C(1-α) = β, which implies that C can be determined only if α = 0 or adjustments are made. Assuming α ≠ 0, the solution simplifies to:

$$P_k = \frac{\beta}{1-\alpha} \tag{5}$$

Thus, the general solution becomes:

$$X_k = \alpha^k H_0 + \frac{\beta}{1-\alpha} \tag{6}$$

To determine the value of H_0, we apply the initial condition. Setting k = 0 yields:

$$1 = X_0 = \alpha^0 H_0 + \frac{\beta}{1-\alpha} \tag{7}$$

which simplifies to:

$$H_0 = 1 - \frac{\beta}{1-\alpha} \tag{8}$$

Substituting H_0back into the general solution provides:

$$X_k = (1 - \frac{\beta}{1-\alpha})\alpha^k + \frac{\beta}{1-\alpha} \tag{9}$$

Notably, solving a first-order linear difference equation requires the specification of a single piece of information, typically an initial or final condition, similar to solving a first-order differential equation (Sendov, 2020).

1.3. Introduction to Second-Order Difference Equation

Second-order linear difference equations extend the first-order approach. Consider the initial value problem defined by:

$$x_{t+1} = ax_{t+1} + bx_t = c_t \tag{10}$$

where a and b are constants, and c_t is a given forcing function. The general solution is composed of both a homogeneous and a particular part:

$$x_t = x_t^H + x_t^P \tag{11}$$

The homogeneous component satisfies the associated homogeneous equation:

$$x_{t+1} + ax_{t+1} + bx_t = 0 \tag{12}$$

Assuming $x_t = \lambda^t$, substitution into the homogeneous equation yields the characteristic (or auxiliary) equation:

$$\lambda^2 + a\lambda + \mathrm{b} = 0 \tag{13}$$

The roots of this quadratic equation determine the form of the homogeneous solution, as will be discussed in subsequent sections.

Meanwhile, the particular solution x_t^P depends on the form of c_t, such as constant, polynomial, or exponential functions. The complete solution is:

$$x_t = x_t^H + x_t^P \tag{14}$$

Unlike first-order cases, second-order difference equations require two initial conditions (e.g., x_0 and x_1) to fully determine the constants. This is analogous to solving second-order differential equations (Hammond, 2020).

While existing literature often focuses on either traditional mesh analysis or continuous-time differential equation modelling of circuits, this study offers a distinct approach by formulating and solving both RL and RLC circuits using discrete-time difference equations, alongside a full analytical derivation for a ladder-type DC mesh network. Unlike prior works, such as those of Salehizadeh & Nouri (2020), which emphasize the pedagogical value in the use of difference equations but lack convergence and complexity comparisons, this paper offers complete recursive solutions, a characteristic equation for ladder circuits, and a count analysis on equations versus mesh methods. Moreover, the use of fixed recurrence structures and boundary condition adjustments across increasing mesh sizes showcases a scalable aspect that is especially suitable for digital simulation environments. This integrated treatment of first- and second-order difference equations across circuit types contributes to enhance the area of discrete-time electrical circuit analysis, not just as a modelling tool but also in broadening one's analytical view.

2. Methodology

This section solves three circuits using the difference equations method. **Figure 1** shows the 4-mesh DC circuit used in this study.

2.1. Solving Mesh Circuit Using Difference Equation

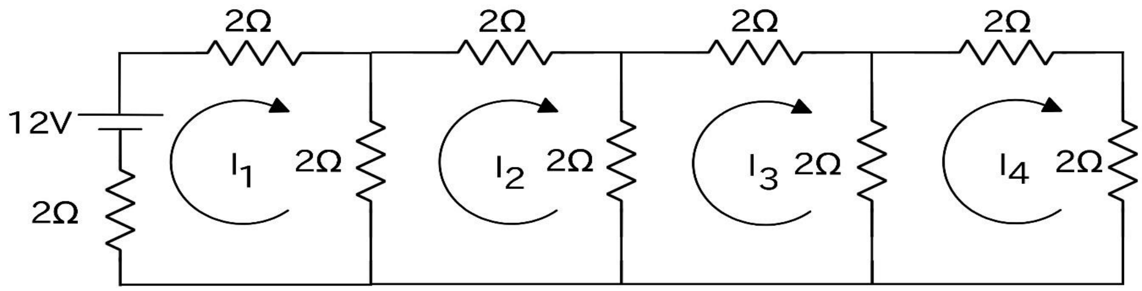

Figure 1. 4-Mesh DC circuit

Ladder circuits like the one above are commonly found in resistive sensor arrays, voltage divider networks, and digital-to-analog conversion hardware. Understanding current distributions in such networks is crucial for accurate signal processing and component protection (Zhang, 2023). The circuit is analyzed using mesh currents I_1 to I_4. Mesh 1 to Mesh 4 correspond to the current loops I_1 through I_4 shown in the circuit diagram.

The circuit difference equation is given by:

$$6I_n - 2(I_{n-1} + I_{n+1}) = 0 \tag{15}$$

Dividing throughout by 2 simplifies it to:

$$3I_n - (I_{n-1} + I_{n+1}) = 0 \tag{16}$$

Rearranging yields the standard form:

$$I_{n+1} = 3I_n - I_{n-1} \tag{17}$$

This is a second-order linear difference equation.

Assume solution:

$$I_n = k\lambda^n \tag{18}$$

Substituting into Equation 17:

$$k\lambda^{n+1} = 3k\lambda^n - k\lambda^{n-1} \tag{19}$$

Shift indices to match powers of λ^n:

$$\lambda^{n+1} = k\lambda^n - \lambda^{n-1} \tag{20}$$

Divide both sides by λ^{n-1}:

$$\lambda^2 = 3\lambda - 1 \tag{21}$$

$$\lambda^2 - 3\lambda + 1 = 0 \tag{22}$$

Using quadratic formula:

$$\lambda = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2} \tag{23}$$

Thus:

$$\lambda_1 = \frac{3+\sqrt{5}}{2}, \quad \lambda_2 = \frac{3-\sqrt{5}}{2} \tag{29}$$

Hence:

$$I_n = A(\frac{3+\sqrt{5}}{2})^n + B(\frac{3-\sqrt{5}}{2})^n \tag{30}$$

Write expressions for $I_1, I_2, I_3, and\ I_4$:

$$I_1 = A(\frac{3+\sqrt{5}}{2})^1 + B(\frac{3-\sqrt{5}}{2})^1 \tag{24}$$

$$I_2 = A(\frac{3+\sqrt{5}}{2})^2 + B(\frac{3-\sqrt{5}}{2})^2 \tag{25}$$

$$I_3 = A(\frac{3+\sqrt{5}}{2})^3 + B(\frac{3-\sqrt{5}}{2})^3 \tag{26}$$

$$I_4 = A(\frac{3+\sqrt{5}}{2})^4 + B(\frac{3-\sqrt{5}}{2})^4 \tag{27}$$

Applying Kirchhoff's Voltage Law (KVL) around Mesh 1:

$$12 = 6I_1 - 2I_2 \tag{28}$$

Applying KVL around Mesh 4:

$$0 = -2I_3 + 6I_4 \tag{29}$$

Substituting $I_1 and\ I_2$ into Equation 28:

$$12 = 6\left[A\left(\frac{3+\sqrt{5}}{2}\right)^1 + B\left(\frac{3-\sqrt{5}}{2}\right)^1\right] - 2[A\left(\frac{3+\sqrt{5}}{2}\right)^2 + B\left(\frac{3-\sqrt{5}}{2}\right)^2] \tag{30}$$

Simplifying:

$$6 = [A\left(\frac{9+3\sqrt{5}}{2}\right) + B\left(\frac{9-3\sqrt{5}}{2}\right)] - [A\left(\frac{7+3\sqrt{5}}{2}\right) + B\left(\frac{7-3\sqrt{5}}{2}\right)] \tag{31}$$

$$6 - A = B \tag{32}$$

Substituting I_3 and I_4 into Equation 29:

$$0 = -2[A\left(\frac{3+\sqrt{5}}{2}\right)^3 + B(\frac{3-\sqrt{5}}{2})^3] + 6[A\left(\frac{3+\sqrt{5}}{2}\right)^4 + B(\frac{3-\sqrt{5}}{2})^4] \tag{33}$$

Simplifying:

$$0 = [A(-18 - 8\sqrt{5}) + B(-18 + 8\sqrt{5})] + [A(141 + 63\sqrt{5}) + 3B(141 - 63\sqrt{5})] \tag{34}$$

$$0 = A(123 + 55\sqrt{5}) + B(123 - 55\sqrt{5}) \tag{35}$$

Solving for A and B using the previously established Equation 32 and Equation 35:

$$0 = A\left(123 + 55\sqrt{5}\right) + (6 - A)\left(123 - 55\sqrt{5}\right) \tag{36}$$

$$A = 3 - \frac{369\sqrt{5}}{275} \tag{37}$$

$$B = 6 - \left(3 - \frac{369\sqrt{5}}{275}\right) = 3 + \frac{369\sqrt{5}}{275} \tag{38}$$

Substituting A and B into I_n:

$$I_n = (3 - \frac{369\sqrt{5}}{275})\left(\frac{3+\sqrt{5}}{2}\right)^n + (3 + \frac{369\sqrt{5}}{275})\left(\frac{3-\sqrt{5}}{2}\right)^n \tag{39}$$

Solving for $I_1, I_2, I_3\ and\ I_4$:

$$I_1 = (3 - \frac{369\sqrt{5}}{275})\left(\frac{3+\sqrt{5}}{2}\right)^1 + (3 + \frac{369\sqrt{5}}{275})\left(\frac{3-\sqrt{5}}{2}\right)^1 = \frac{126}{55}A \tag{40}$$

$$I_2 = \left(3 - \frac{369\sqrt{5}}{275}\right)\left(\frac{3+\sqrt{5}}{2}\right)^2 + \left(3 + \frac{369\sqrt{5}}{275}\right)\left(\frac{3-\sqrt{5}}{2}\right)^2 = \frac{48}{55}A \tag{41}$$

$$I_3 = (3 - \frac{369\sqrt{5}}{275})\left(\frac{3+\sqrt{5}}{2}\right)^3 + (3 + \frac{369\sqrt{5}}{275})\left(\frac{3-\sqrt{5}}{2}\right)^3 = \frac{18}{55}A \tag{42}$$

$$I_4 = (3 - \frac{369\sqrt{5}}{275})\left(\frac{3+\sqrt{5}}{2}\right)^4 + (3 + \frac{369\sqrt{5}}{275})\left(\frac{3-\sqrt{5}}{2}\right)^4 = \frac{6}{55}A \tag{43}$$

2.1.1. Efficiency Comparison with Mesh Analysis

To evaluate the efficiency of the method, both difference equation and traditional mesh analysis were applied to solve uniform 4-mesh, 5-mesh, and up to 7-mesh DC ladder circuits. While this paper presents the full derivation only for the 4-mesh case using difference equations, mesh analysis was also applied to each configuration for comparison in terms of computational effort and equation count, as shown in **Table 1**. The focus is to show the scaling of each method with increasing circuit size.

Table 1. Equation count for increasing mesh

Configuration	No. of Equations (Mesh Analysis)	No. of Equations (Difference Equation)
4-Mesh	17	25
5-Mesh	22	17
6-Mesh	27	19
7-Mesh	32	21

In this context, we define complexity as the total number of symbolic equations to be solved, which represents the algebraic workload associated with modelling and solving the circuit. Compared to mesh analysis, which grows by five equations per added mesh, the difference equation method increases by only two, using a fixed recursive structure with adjustable boundary conditions. Initially, the 4-mesh configurations needed 25 equations from the full derivation, but once the recurrence relation was established, it could be applied to higher mesh sizes, ultimately reducing the total number to 17 for the 5-mesh case, increasing thereafter by only two equations. From higher-mesh configuration onward, it yields a significantly lower total equation count, thus making it more efficient and suited for large or infinite ladder circuits.

As discussed by Green (2024) and Salehizadeh & Nouri (2020), varying initial conditions within the fixed recurrence structure reduces algebraic effort and supports a scalable modelling strategy for larger systems.

2.2. Solving RL Circuit Using Difference Equation

The circuit shown in **Figure 2** represents a basic RL circuit driven by a time-varying voltage source V(t), with a resistor of R = 2 Ω, an inductor of L = 1 H, an initial condition of current = 0, a time step of 0.01 s, and a unit step input V(k) = 1.

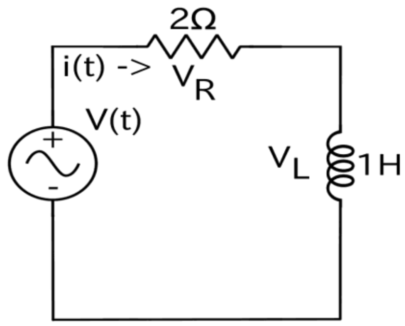

Figure 2. A simple RL circuit

The values $R = 2\,\Omega$ and $L = 1\,H$ were chosen to reflect realistic inductive load scenarios such as electromagnetic relays or switching transformers. These values allow the model to emulate slow current rise conditions found in practical DC applications, such as motor start-up behaviour and inrush current limiting.

To formulate the dynamic behaviour of the circuit, Kirchhoff's Voltage Law (KVL) is applied:

$$V(t) = V_R(t) + V_L(t) \tag{44}$$

The voltage drops across the resistor and inductor is expressed as:

$$V(t) = Ri(t) + L\frac{di(t)}{dt} \tag{45}$$

This equation represents a first-order linear differential equation. To analyse this in a discrete-time simulation, it is converted into a difference equation.

Let the time domain be discretized into small steps of width Δt and define $t = k\Delta t$, where $k = 0,1,2,3,\ldots,N$. The derivative is approximated using a backward difference method:

$$\frac{di(t)}{dt} \approx \frac{i(k+1) - i(k)}{\Delta \mathrm{t}} \tag{46}$$

Substitute this into:

$$V(t) = Ri(t) + L\frac{i(k+1) - i(k)}{\Delta \mathrm{t}} \tag{47}$$

Rearranging to solve for i(k+1):

$$i(k+1) = (1 - \frac{R\Delta \mathrm{t}}{L})i(k) + \frac{\Delta \mathrm{t}}{\mathrm{L}}V(k) \tag{48}$$

This is the first-order difference equation describing the RL circuit.

Substituting the values:

$$i(k+1) = (1 - \frac{2(0.01)}{1})i(k) + \frac{0.01}{1}V(k) \tag{49}$$

$$i(k+1) = 0.98i(k) + 0.01V(k) \quad (50)$$

Solving for discrete currents by substituting k = 0, 1, 2, 3, … N.

$$i(0+1) = 0.98i(0) + 0.01V(0) \quad (51)$$

$$i(1) = 0.98(0) + 0.01(1) \quad (52)$$

$$i(1) = 0.01\ A \quad (53)$$

$$i(1+1) = 0.98i(1) + 0.01V(1) \quad (54)$$

$$i(2) = 0.98(0.01) + 0.01(1) \quad (55)$$

$$i(2) = 0.0198\ A \quad (56)$$

$$i(2+1) = 0.98i(2) + 0.01V(2) \quad (57)$$

$$i(3) = 0.98(0.0198) + 0.01(1) \quad (58)$$

$$i(3) = 0.029404\ A \quad (59)$$

$$i(3+1) = 0.98i(3) + 0.01V(3) \quad (60)$$

$$i(4) = 0.98(0.029404) + 0.01(1) \quad (61)$$

$$i(4) = 0.03881592\ A \quad (62)$$

Checking the differences between each step:

$$\mathrm{i}(1) - \mathrm{i}(0) = 0.01 - 0 = 0.01 \quad (63)$$

$$\mathrm{i}(2) - \mathrm{i}(1) = 0.0198 - 0.01 = 0.0098 \quad (64)$$

$$\mathrm{i}(3) - \mathrm{i}(2) = 0.029404 - 0.0198 = 0.009604 \quad (65)$$

$$\mathrm{i}(4) - \mathrm{i}(3) = 0.03881592 - 0.029404 = 0.0941192 \quad (66)$$

To illustrate the transient response of the RL circuit, the current values computed from the first-order difference equation were plotted over time. As shown in **Figure 3**, the graph represents the gradual increase in current where the inductor initially resists dramatic changes until the system stabilizes because of the constant step input.

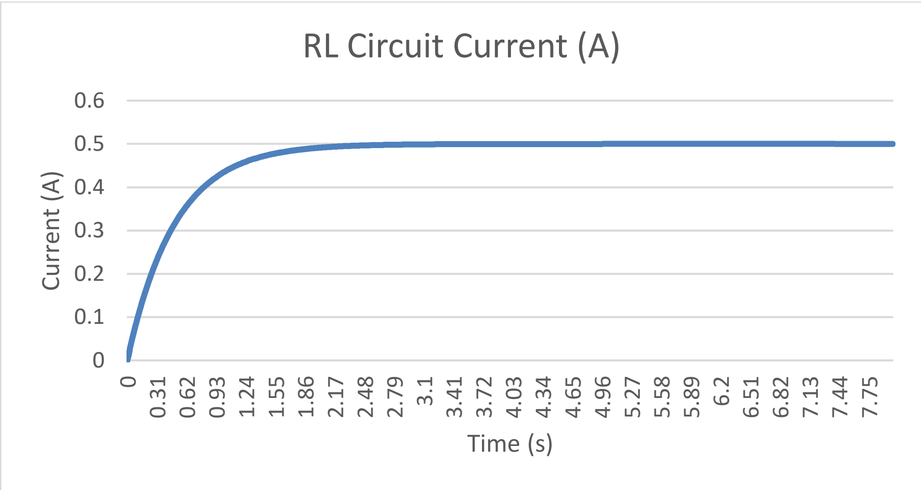

Figure 3. Time response of current in an RL circuit using difference equation

2.3. Solving RLC Circuit Using Difference Equation

The circuit shown in **Figure 4** represents a second-order RLC circuit driven by a time-varying voltage source V(t), with a resistor of $R = 2\,\Omega$, an inductor of $L = 1\,H$, and a capacitor of $C = 0.01\,F$. The initial conditions are set to zero current $i(0) = 0$ and zero past current $i(-1) = 0$, with a time step of $\Delta t = 0.0001\,s$ and a unit step input $V(k) = 1$.

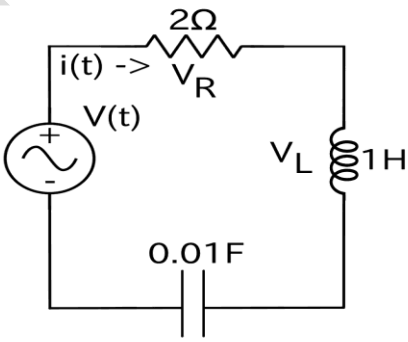

Figure 4. A simple RLC circuit

By choosing $R = 2\,\Omega$, $L = 1\,H$, and $C = 0.01\,F$, the circuit is very similar to real-world filter circuits and tuned resonant systems. This arrangement makes it possible to analyse damped oscillations, which are also observed in power electronics and communication filters.

To analyse the transient behaviour of the circuit, we apply Kirchhoff's Voltage Law (KVL):

$$V(t) = V_R(t) + V_L(t) + V_C(t) \tag{67}$$

The voltage across each component is expressed as:

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \tag{68}$$

Differentiating both sides gives:

$$\frac{dV}{dt} = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i(t)}{C} \tag{69}$$

This leads to the second-order linear differential equation:

$$\frac{dV}{dt} = L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i(t) \tag{70}$$

To analyse this in discrete time, we convert it into a difference equation. Using the backward difference approximations:

$$\frac{di(t)}{dt} \approx \frac{i(k) - i(k-1)}{\Delta\mathrm{t}} \tag{71}$$

$$\frac{d^2i}{dt^2} \approx \frac{i(k+1) - 2i(k) + i(k-1)}{\Delta\mathrm{t}^2} \tag{72}$$

Substitute into the differential equation:

$$\frac{V(k) - V(k-1)}{\Delta\mathrm{t}} = L\frac{i(k+1) - 2i(k) + i(k-1)}{\Delta\mathrm{t}^2} + R\frac{i(k) - i(k-1)}{\Delta\mathrm{t}} + \frac{1}{C}i(k) \tag{73}$$

Multiply through by Δt^2 to eliminate denominators:

$$[V(k) - V(k-1)]\Delta\mathrm{t} = L[i(k+1) - 2i(k) + i(k-1)] + R[i(k) - i(k-1)]\Delta\mathrm{t} + \frac{\Delta\mathrm{t}^2}{C}i(k)$$

Simplifying and grouping like terms:

$$[V(k) - V(k-1)]\Delta = Li(k+1) - 2Li(k) + Li(k-1) + R[i(k) - i(k-1)]\Delta\mathrm{t} + \frac{\Delta\mathrm{t}^2}{C}i(k)$$

$$Li(k+1) = [V(k) - V(k-1)]\Delta t + \left[2L - R\Delta t + \frac{\Delta t^2}{C}\right] i(k) - (L - R\Delta t)i(k-1)$$

Substituting the given values:

L = 1 H, R = 2 Ω, C = 0.01 F, and Δt = 0.0001

This simplifies to:

$$i(k+1) = [V(k) - V(k-1)]0.0001 + \left[2(1) - 2(0.0001) + \frac{(0.0001)^2}{0.01}\right] i(k) - [1 - 2(0.0001)]i(k-1)$$

$$i(k+1) = [V(k) - V(k-1)]0.0001 + (1.99801)i(k) - (0.9998)i(k-1)$$

Since the input is a unit step:

$$V(k) = 1, V(k) \geq 0 \rightarrow V(k) - V(k-1) = \begin{cases} 1, k = 0 \\ 0, k \geq 1 \end{cases}$$

Solving for discrete currents by substituting k = 0, 1, 2, … N.

$$i(0+1) = [V(0) - V(0-1)]0.0001 + (1.99801)i(0) - (0.9998)i(0-1) \quad (74)$$

$$i(1) = 1 + (1.99801)(0) - (0.9998)(0) \quad (75)$$

$$i(1) = 0.0002 \text{ A} \quad (76)$$

$$i(1+1) = [V(1) - V(1-1)]0.0001 + (1.99801)i(1) - (0.9998)i(1-1) \quad (77)$$

$$i(2) = 0 + (1.99801)(0.0002) - (0.9998)(0) \quad (78)$$

$$i(2) = 0.000399602 \text{ A} \quad (79)$$

$$i(2+1) = [V(2) - V(2-1)]0.0001 + (1.99801)i(2) - (0.9998)i(2-1) \quad (80)$$

$$i(3) = 0 + (1.99801)(0.000399602) - (0.9998)(0.0002) \quad (81)$$

$$i(3) = 0.00059844879 \text{ A} \quad (82)$$

$$i(3+1) = [V(3) - V(3-1)]0.0001 + (1.99801)i(3) - (0.9998)\mathrm{i}(3-1) \tag{83}$$

$$i(4) = 0 + (1.99801)(0.00059844879) - (0.9998)(0.000399602) \tag{84}$$

$$i(4) = 0.00079618459\text{ A} \tag{85}$$

Looking at the differences between each step:

$$\mathrm{i}(1) - \mathrm{i}(0) = 0.0002 - 0 = 0.0002 \tag{86}$$

$$\mathrm{i}(2) - \mathrm{i}(1) = 0.000399602 - 0.0002 = 0.000199602 \tag{87}$$

$$\mathrm{i}(3) - \mathrm{i}(2) = 0.00059844879 - 0.000399602 = 0.00019884679 \tag{88}$$

$$\mathrm{i}(4) - \mathrm{i}(3) = 0.00079618459 - 0.00059844879 = 0.0001977358 \tag{89}$$

The current response of the RLC circuit was plotted based on the second-order difference equation (**Figure 5**).

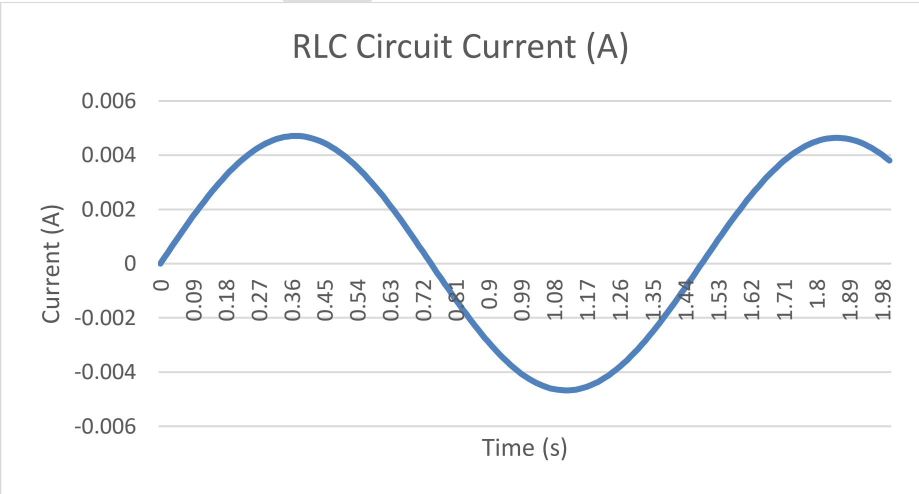

Figure 5. Transient current response in an RLC circuit using difference equation

The graph shows a continuous increase in current with oscillations, reflecting the transient behaviour of the circuit. The inductor resists rapid changes in current, the capacitor stores energy, and the resistor dissipates energy, all of which lead to a steady state.

3. Results and Discussion

Analyzing three distinct circuits using difference equation includes a 4-mesh DC circuit, an RL circuit, and an RLC circuit. The approach uses initial conditions and recurrence relations to compute as illustrated by Salehizadeh & Nouri (2020). The difference equation method is simpler as opposed to the traditional mesh analysis method that uses simultaneous equations (Testbook, 2025). The values of current were provided using characteristic equations in closed form to obtain general solutions (Stevic, 2024).

When stable currents were obtained through the mesh circuit, they fall within the theoretical mesh analysis. As the circuit was switched into the RL circuit, the current exhibited smooth increase as the inductor does not like sudden changes and stabilizes when there is constant input. There was damped oscillation of the RLC circuit which came to rest at the steady-state because of the interaction of the inductor and the capacitor. The different initial conditions presented by Green (2024) are a demonstration of how difference equations do not require the laborious algebraic work as well as they scale in complicated systems.

Applied in practice the technique is useful in modifying the resonant frequency of RLC filters used in communication systems and in examining the transients in power electronic systems using the second order difference equations. Its effectiveness will favor more realistic modeling and configuration of circuits on digital platforms.

The ladder, RL, and RLC circuits all demonstrate stable and convergent behaviour. A second-order recurrence relation is solved for the ladder circuit through a characteristic equation, with the resulting mesh currents converging as expected through the four meshes. The first-order difference equation model for the RL circuit with $\alpha = 0.98$ approaches steady state gradually, while the RLC circuit remains stable since the roots of its characteristic equation fall inside the unit circle. Small numerical errors from the backward difference approximation are negligible on the grounds of chosen time steps to maintain accuracy and stability throughout all of the simulations.

4. Conclusion

This paper has shown how the dynamic behaviour of an electrical circuit is well represented by using difference equations and how the following circuits, a 4-mesh DC ladder, an RL circuit and an RLC circuit can be modelled. When step inputs of unit strength were applied, the responses were consistent with the expected theoretical responses of steady-state current flows in the mesh circuit appropriate to resistive sensor arrays and voltage dividers; smooth transitions in steady-state with the RL circuit reflecting real world inductive loads, such as starting motors and relays; and damped oscillations in the RLC circuit applicable to resonant filters and power conversion. Analysis of different methods of efficiency indicated that although the 4-mesh system needed 25 equations, the recurrence relations form yielded fewer equations with larger mesh systems, growing only two per additional mesh-as compared to the traditional form of solving mesh analysis which has five more equations per extra mesh-that it could be scaled more easily and be applied to more ladder networks of a larger or even infinite size. The above findings indicate the usefulness of difference equations in modelling and designing the contemporary electrical systems. Future works include to apply difference equations to complex circuits, which are computationally intensive when using conventional methods like mesh analysis; this approach may provide an effective solution; and focus on the application of difference equation to AC circuit analysis, transient phenomena in operational systems, and digital control of electronic power.

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Conflicts of Interest

The authors declare no conflict of interest.

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